**Chapter 7: Matrices and Determinants**

7.3 – The Inverse of a Square Matrix

In the real number system, the multiplicative inverse of is because \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

The inverse of a number can be denoted as \_\_\_\_\_\_\_\_\_\_.

The definition of the multiplicative inverse of a matrix is similar.

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DEFINITION OF THE INVERSE OF A SQUARE MATRIX

Let be an matrix. If there exists matrix such that:

is called the **inverse** of

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Example 1: Given the following matrices

and

a) Find .

b) Find

What do you notice?

If a matrix has an inverse, is called \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. A non-square matrix cannot have an inverse.

To see this, let’s say is of order and is of order .

Write the order of : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Write the order of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Since the products of and are of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ orders, they cannot be equal to each other.

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Example 2: Given the following matrices

and .

a) Find .

b) Find

Therefore \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Example 3: Given the following matrices

and

a) Find .

b) Find

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To find the inverse of a matrix we must first calculate the **determinant** of the matrix.

If , then the determinant is equal to \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

\*\*Matrix is invertible if and only if \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .\*\*

Example 4: Find the determinant of the following matrices.

a) b) c)

Using the Determinant to find the Inverse of a Matrix

If , then .

Example 5: If possible, find the inverse of the following matrices.

a)

b)

c)

d)

USING THE CALCULATOR TO FIND AN INVERSE MATRIX

USING INVERSES TO SOLVE A SYSTEM OF LINEAR EQUATIONS

You know that a system of linear equations can have exactly one solution, infinitely many solutions, or no solution. If a coefficient matrix of a *square* system is invertible, the system has a unique solution.

If is an invertible matrix, the system of linear equations represented by has a unique solution given by .

Let’s take the following system and write it as a COEFFICIENT MATRIX.

Use an inverse matrix to solve the system of linear equations.

a)

b)

c)