**Chapter 7: Matrices and Determinants**

7.3 – The Inverse of a Square Matrix

In the real number system, the multiplicative inverse of $a$ is $\frac{1}{a}$ because \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

The inverse of a number can be denoted as \_\_\_\_\_\_\_\_\_\_.

The definition of the multiplicative inverse of a matrix is similar.

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DEFINITION OF THE INVERSE OF A SQUARE MATRIX

Let $A$ be an $n ×n$ matrix. If there exists matrix $A^{-1}$ such that:

$$AA^{-1}=I\_{n}=A^{-1}A$$

$A^{-1} $is called the **inverse** of $A.$

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Example 1: Given the following matrices

$A=\left[\begin{matrix}-1&2\\-1&1\end{matrix}\right] $ and $B=\left[\begin{matrix}1&-2\\1&-1\end{matrix}\right].$

a) Find $AB$.

b) Find $BA.$

What do you notice?

If a matrix $A$ has an inverse, $A$ is called \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. A non-square matrix cannot have an inverse.

To see this, let’s say $A$ is of order $m ×n$ and $B$ is of order $n ×m$.

Write the order of $AB$: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Write the order of $BA:$ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Since the products of $AB$ and $BA$ are of \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ orders, they cannot be equal to each other.

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Example 2: Given the following matrices

$A=\left[\begin{matrix}1&4\\-1&-3\end{matrix}\right] $ and $B=\left[\begin{matrix}-3&-4\\1&1\end{matrix}\right]$.

a) Find $AB$.

b) Find $BA.$

Therefore \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Example 3: Given the following matrices

$A= \left[\begin{matrix} 2&-17& 11\\-1& 11&-7\\ 0& 3&-2\end{matrix}\right] $and $B=\left[\begin{matrix}1& 1& 2\\2& 4&-3\\3& 6&-5\end{matrix}\right]$

a) Find $AB$.

b) Find $BA.$

-----------------------------------------------------------------------------------------------------------------------------Finding the Inverse of a $2 × 2$ Matrix

To find the inverse of a $2 ×2 $matrix we must first calculate the **determinant** of the matrix.

If $A=\left[\begin{matrix}a&b\\c&d\end{matrix}\right]$, then the determinant is equal to \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

\*\*Matrix $A$ is invertible if and only if \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ $\ne 0$.\*\*

Example 4: Find the determinant of the following matrices.

a) $A=\left[ \begin{matrix} 3&-1\\-2& 2\end{matrix}\right]$ b) $B=\left[\begin{matrix} 3&-1\\-6& 2\end{matrix}\right]$ c) $C=\left[\begin{matrix}2&5\\1&3\end{matrix}\right]$

Using the Determinant to find the Inverse of a $2 × 2$ Matrix

If $A=\left[\begin{matrix}a&b\\c&d\end{matrix}\right]$, then $A^{-1}=\frac{1}{ad-bc}\left[\begin{matrix}d&-b\\-c&a\end{matrix}\right]$.

Example 5: If possible, find the inverse of the following matrices.

a) $A=\left[\begin{matrix} 3&-1\\-2& 2\end{matrix}\right]$

b) $B=\left[\begin{matrix}1&2\\3&4\end{matrix}\right]$

c) $C=\left[\begin{matrix}2&4\\4&8\end{matrix}\right]$

d) $D=\left[\begin{matrix}11&1\\-1&0\end{matrix}\right]$

USING THE CALCULATOR TO FIND AN INVERSE MATRIX

$A=\left[\begin{matrix}-3&2\\ 7&4\end{matrix}\right]$ $B=\left[\begin{matrix}1&-4&2\\2&-9&5\\1&-5&4\end{matrix}\right]$ $C=\left[\begin{matrix}3&1&0\\1&1&1\\1&-1&2\end{matrix}\right]$

$A^{-1}=$ $B^{-1}=$ $C^{-1}=$

USING INVERSES TO SOLVE A SYSTEM OF LINEAR EQUATIONS

You know that a system of linear equations can have exactly one solution, infinitely many solutions, or no solution. If a coefficient matrix $A$ of a *square* system is invertible, the system has a unique solution.

If $A$ is an invertible matrix, the system of linear equations represented by $AX=B$ has a unique solution given by $X=A^{-1}B$.

Let’s take the following system and write it as a COEFFICIENT MATRIX.

$$\begin{matrix}2x+3y+z =-1\\3x+3y+z=1\\2x+4y+z =-2\end{matrix}$$

Use an inverse matrix to solve the system of linear equations.

a) $x-2y=5$

 $2x-3y=10$

b) $x+y+z=0$

 $3x+5y+4z=5$

 $3x+6y+5z=2$

c) $7x-3y +2w=41$

 $-2x+ y -w=-13$

 $4x +z-2w=12$

$$ -x+y -w=-8$$